Assignment on Real Numbers
Class X(10th)
CBSC Board

Theorem (Euclid's Division Lemma) : Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, $0 \le r < b$.

Question: 01: Find the integers q and r for the following pairs of positive integers a and b?
(I) 20, 5 (II) 15, 7 (III) 75,25 (IV) 49, 8 (V) 96,11

Euclid's division algorithm:
To obtain the HCF of two positive integers, say c and d, with $c > d$, follow the steps below:
Step 1 : Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.
Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.
Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.
This algorithm works because HCF (c, d) = HCF (d, r) where the symbol HCF (c, d) denotes the HCF of c and d, etc.

Let us see how the algorithm works, through an example first. Suppose we need to find the HCF of the integers 474 and 36. We start with the larger integer, that is, 474. Then we use Euclid's lemma to get

$$474 = 36 \times 13 + 6$$

Now consider the divisor 36 and the remainder 6, and apply the division lemma to get

$$36 = 6 \times 6 + 0$$

Notice that the remainder has become zero, and we cannot proceed any further. Now we can claim that the HCF of 474 and 36 is the divisor at this stage, i.e., 6. You can easily also verify this by listing all the factors of 474 and 36.

Question: 02: Use Euclid's algorithm to find the HCF of the following pairs:
(I) 4062 and 82576 (II) 68539 and 98564 (III) 85621and 12364

Question: 03: Show that every positive even integer is of the form 2q, and that every positive odd integer is of the form 2q + 1, where q is some integer.

Question: 04: Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where q is some integer.

Question: 05: Show that any positive odd integer is of the form 8q + 1, or 8q + 3, or 8q + 5, or 8q+7 where q is some integer.

Question: 06: Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m. [Hint : Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.]

Question: 07: . Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Question: 08: Use Euclid's division lemma to show that the quadruple of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Question: 09: Two numbers are in the ratio of 13:11. If their H.C.F. is 13, find the numbers.

Question: 10: Write the prime factorization of the greatest 3-digit number.($Hint : 999 = 3^3 * 37$)

The prime factorisation of a natural number is unique, except for the order of its factors.
In general, given a composite number x, we factorise it as x = $p_1 p_2 p_3 \ldots\ldots\ldots\ldots p_n$, where $p_1, p_2, p_3, \ldots\ldots\ldots\ldots p_n$ are primes and written in ascending order, i.e., $p_1 \leqslant p_2 \leqslant p_3 \ldots\ldots\ldots\ldots \leqslant p_n$ If we combine the same primes, we will get powers of primes.

For any two positive integers a and b, HCF (a, b) × LCM (a, b) = a × b. We can use this result to find the LCM of two positive integers, if we have already found the HCF of the two positive integers.
Note that:

- **HCF(a, b) = Product of the smallest power of each common prime factor in the numbers.**
- **LCM (a, b) = Product of the greatest power of each prime factor, involved in the numbers.**

Question: 11: Consider the numbers 4^n , where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Question: 12: Find the LCM and HCF of following pair by the prime factorisation method.
(I) 6 and 20 (II) 96 and 404 (III) 26 and 91 (IV) 34 and 119

Question: 13: Given that HCF (306, 1314) = 18, find LCM (306, 1314).

Question: 14: On a circular path man A completes in 12 minutes and man B completes in 16 minutes one round of the circular path. Suppose both start at the same time, same point and the same direction, find the time when they meet at the starting point.

Theorem: Let p be a prime number. If p divides a^2 , then p divides a , where a is a positive integer.
Proof : Let the prime factorisation of a be as follows :
$a=p_1 p_2 \ldots\ldots\ldots\ldots\ldots p_n$ where $p_1, p_2, p_3 \ldots\ldots\ldots\ldots\ldots p_n$ are primes, not necessarily distinct.
Therefore, $a^2=(p_1 p_2 \ldots\ldots\ldots\ldots\ldots p_n)(p_1 p_2 \ldots\ldots\ldots\ldots\ldots p_n)=p_1^2 p_2^2 p_3^2 \ldots\ldots\ldots\ldots p_n^2$
Now, we are given that p divides a^2 . Therefore, from the Fundamental Theorem of Arithmetic, it follows that p is one of the prime factors of a^2 . However, using the uniqueness part of the Fundamental Theorem of Arithmetic, we realise that the only prime factors of a^2 are $p_1, p_2, p_3 \ldots\ldots\ldots\ldots\ldots p_n$. So p is one of $p_1, p_2, p_3 \ldots\ldots\ldots\ldots\ldots p_n$.
Now, since $a=p_1 p_2 \ldots\ldots\ldots\ldots\ldots p_n$, p divides a .

Question: 15: prove the following as irrational numbers. $\sqrt{2}, \sqrt{3}, 2-\sqrt{3}, 4\sqrt{3}, \frac{1}{\sqrt{3}}$.

Theorem : Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers.

Theorem : Let $x=\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

Theorem : Let $x=\frac{p}{q}$ be a rational number, such that the prime factorisationof q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

Answers:

1. (I) 4,0 (II)2,1 (III) 3,0 (iv) 6,1 (v) 8,8

2. (I) 6 (II) 1 (II)1

9. 136, 143

10. $Hint: 999 = 3^3 * 37$

11. No

13. 22338

14. 48 minutes

For more detailed solution please mail us- skdwivedi2009@gmail.com