# Matrices

## Solution of Exercise 3.1:

**Question 1:**

**(i)** In the given matrix, the number of rows is 3 and the number of columns is 4.

Therefore, the order of the matrix is $3 \times 4$.

(ii) Since the order of the matrix is $3 \times 4$,

Therefore the total elements are $3 \times 4 = 12$.

(iii) $a_{13} =$ Element of matrix belongs to first row and third column is 19.

$a_{21} =$ Element of matrix, belongs to second row and first column is 35.

$a_{33} =$ Element of matrix, belongs to third row and third column is -5.

$a_{24} =$ Element of matrix, belongs to second row and fourth column is 12.

$a_{23} =$ Element of matrix, belongs to second row and third column is $\frac{5}{2}$.

**Question 2:**

As we know that if a matrix is of order $m \times n$, it has $mn$ elements. Thus, to find the all possible solution of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are (1,24), (24,1), (2,12), (12,2), (3,8), (8,3), (4,6) and (6,4).

Therefore the possible orders of a matrix are:
$1 \times 24,\ 24 \times 1,\ 2 \times 12,\ 12 \times 1,\ 3 \times 8,\ 8 \times 3,\ 4 \times 6,\ 6 \times 4$

If matrix has 13 elements then the possible orders are $1 \times 13,\ 13 \times 1$.

**Question 3:**

As we know that if a matrix is of order $m \times n$, it has $mn$ elements. Thus, to find the all possible solution of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are (1,18), (18,1), (2,9), (9,2), (3,6) and (6,3).

Therefore the possible orders of a matrix are: $1\times18$, $18\times1$, $2\times9$, $9\times1$, $3\times6$, $6\times3$

If matrix has 5 elements then the possible orders are $1\times5$, $5\times1$.

**Question 4:**

(i) In general, $2\times2$ matrix is given by $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

It is given that $a_{ij} = \dfrac{(i+j)^2}{2}$

Therefore, $a_{11} = \dfrac{(1+1)^2}{2} = 2$, $a_{12} = \dfrac{(1+2)^2}{2} = \dfrac{9}{2}$, $a_{21} = \dfrac{(2+1)^2}{2} = \dfrac{9}{2}$ and $a_{22} = \dfrac{(2+2)^2}{2} = 8$.

Hence $A=\begin{bmatrix} 2 & \dfrac{9}{2} \\ \dfrac{9}{2} & 8 \end{bmatrix}$

(ii) In general, $2\times2$ matrix is given by $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

It is given that $a_{ij} = \dfrac{i}{j}$

Therefore, $a_{11} = \dfrac{1}{1} = 1$, $a_{12} = \dfrac{1}{2}$, $a_{21} = \dfrac{2}{1} = 2$ and $a_{22} = \dfrac{2}{2} = 1$.

Hence $A=\begin{bmatrix} 1 & \dfrac{1}{2} \\ 2 & 1 \end{bmatrix}$

(iii) In general, $2\times2$ matrix is given by $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

It is given that $a_{ij} = \dfrac{(i+2j)^2}{2}$

Therefore, $a_{11}=\frac{(1+2\times1)^2}{2}=\frac{9}{2}$, $a_{12}=\frac{(1+2\times2)^2}{2}=\frac{25}{2}$, $a_{21}=\frac{(2+2\times1)^2}{2}=8$ and $a_{22}=\frac{(2+2\times2)^2}{2}=18$.

Hence A= $\begin{bmatrix}\frac{9}{2} & \frac{25}{2}\\ 8 & 18\end{bmatrix}$

**Question 5:**

(i) In general, $3\times4$ matrix is given by A= $\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}$

It is given that $a_{ij}=\frac{1}{2}\left|-3i+j\right|$

Therefore, $a_{11}=\frac{\left|-3\times1+1\right|}{2}=\frac{\left|-2\right|}{2}=1$, $a_{12}=\frac{\left|-3\times1+2\right|}{2}=\frac{\left|-1\right|}{2}=\frac{1}{2}$,

$a_{13}=\frac{\left|-3\times1+3\right|}{2}=\frac{\left|-0\right|}{2}=0$, $a_{14}=\frac{\left|-3\times1+4\right|}{2}=\frac{\left|1\right|}{2}=\frac{1}{2}$, $a_{21}=\frac{\left|-3\times2+1\right|}{2}=\frac{\left|-5\right|}{2}=\frac{5}{2}$,

$a_{22}=\frac{\left|-3\times2+2\right|}{2}=\frac{\left|-4\right|}{2}=2$, $a_{23}=\frac{\left|-3\times2+3\right|}{2}=\frac{\left|-3\right|}{2}=\frac{3}{2}$, $a_{24}=\frac{\left|-3\times2+4\right|}{2}=\frac{\left|-2\right|}{2}=1$,

$a_{31}=\frac{\left|-3\times3+1\right|}{2}=\frac{\left|-8\right|}{2}=4$, $a_{32}=\frac{\left|-3\times3+2\right|}{2}=\frac{\left|-7\right|}{2}=\frac{7}{2}$, $a_{33}=\frac{\left|-3\times3+3\right|}{2}=\frac{\left|-6\right|}{2}=3$ and

$a_{34}=\frac{\left|-3\times3+4\right|}{2}=\frac{\left|-5\right|}{2}=\frac{5}{2}$.

Hence A= $\begin{bmatrix}1 & \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{5}{2} & 2 & \frac{3}{2} & 1\\ 4 & \frac{7}{2} & 3 & \frac{5}{2}\end{bmatrix}$

(ii) In general, $3\times4$ matrix is given by A= $\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ a_{31} & a_{32} & a_{33} & a_{34}\end{bmatrix}$

It is given that $a_{ij}=2i-j$

Therefore, $a_{11} = 2\times1-1=1$, $a_{12} = 2\times1-2=0$, $a_{13} = 2\times1-3=-1$, $a_{14} = 2\times1-4=-2$, $a_{21} = 2\times2-1=3$, $a_{22} = 2\times2-2=2$, $a_{23} = 2\times2-3=1$, $a_{24} = 2\times2-4=0$, $a_{31} = 2\times3-1=5$, $a_{32} = 2\times3-2=4$, $a_{33} = 2\times3-3=3$ and $a_{34} = 2\times3-4=2$.

Hence $A=\begin{bmatrix}1 & 0 & -1 & -2\\ 3 & 2 & 1 & 0\\ 5 & 4 & 3 & 2\end{bmatrix}$

**Question 6:**

(i) As given the matrices are equal, therefore each elements of corresponding matrix are equals.

$$\begin{bmatrix}4 & 3\\ x & 5\end{bmatrix}=\begin{bmatrix}y & z\\ 1 & 5\end{bmatrix}$$

Hence x=1, y=4 and z=3.

(ii) As given the matrices are equal, therefore each elements of corresponding matrix are equals.

$$\begin{bmatrix}x+y & 2\\ 5+z & xy\end{bmatrix}=\begin{bmatrix}6 & 2\\ 5 & 8\end{bmatrix}$$

We get following relations:

$x+y=6, xy=8, 5+z=5$

Therefore z=0,

Solving $x+y=6, xy=8$

$$x+\frac{8}{x}=6$$
$$x^2-6x+8=0$$
$$(x-4)(x-2)=0$$
$$x=2,4$$
$$y=4,2$$

Two solutions one is $x=2, y=4, z=0$ and $x=4, y=2, z=0$

(iii) As given the matrices are equal, therefore each elements of corresponding matrix are equals.

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

We get following relations:

$x+y+z=9, x+z=5, y+z=7$

On solving above equations

Hence x=2, y=4 and z=3.

**Question 7:**

As given the matrices are equal, therefore each elements of corresponding matrix are equals.

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

We get following relations:

$a-b=-1, 2a+c=5, 2a-b=0, 3c+d=13$

On solving above equations

Hence a=1, b=2, c=3 and d=4.

**Question 8:**

**Answer(C).** It is known that a given matrix is square matrix only when number of rows and number of columns are equal.

**Question 9:**

As given the matrices are equal, therefore each elements of corresponding matrix are equals.

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

We get following relations:

$3x+7=0, y-2=5, y+1=8, 2-3x=4$

On solving above equations

$x = -\frac{7}{3}, -\frac{2}{3}$

$y = 7$

We get two different value of x, which is not true for matrices given equal.

Answer (B)

**Question 10:**

The given matrix of the order $3 \times 3$ has 9 elements.

Each of the elements can take value 0 or 1, so each element has two options.

Using multiplication method, the required number of matrices is $2^9 = 512$

Answer (D)